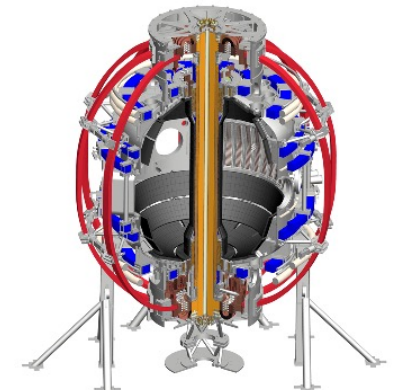


# Modes in the GAE 2<sup>nd</sup> Harmonic Frequency Band

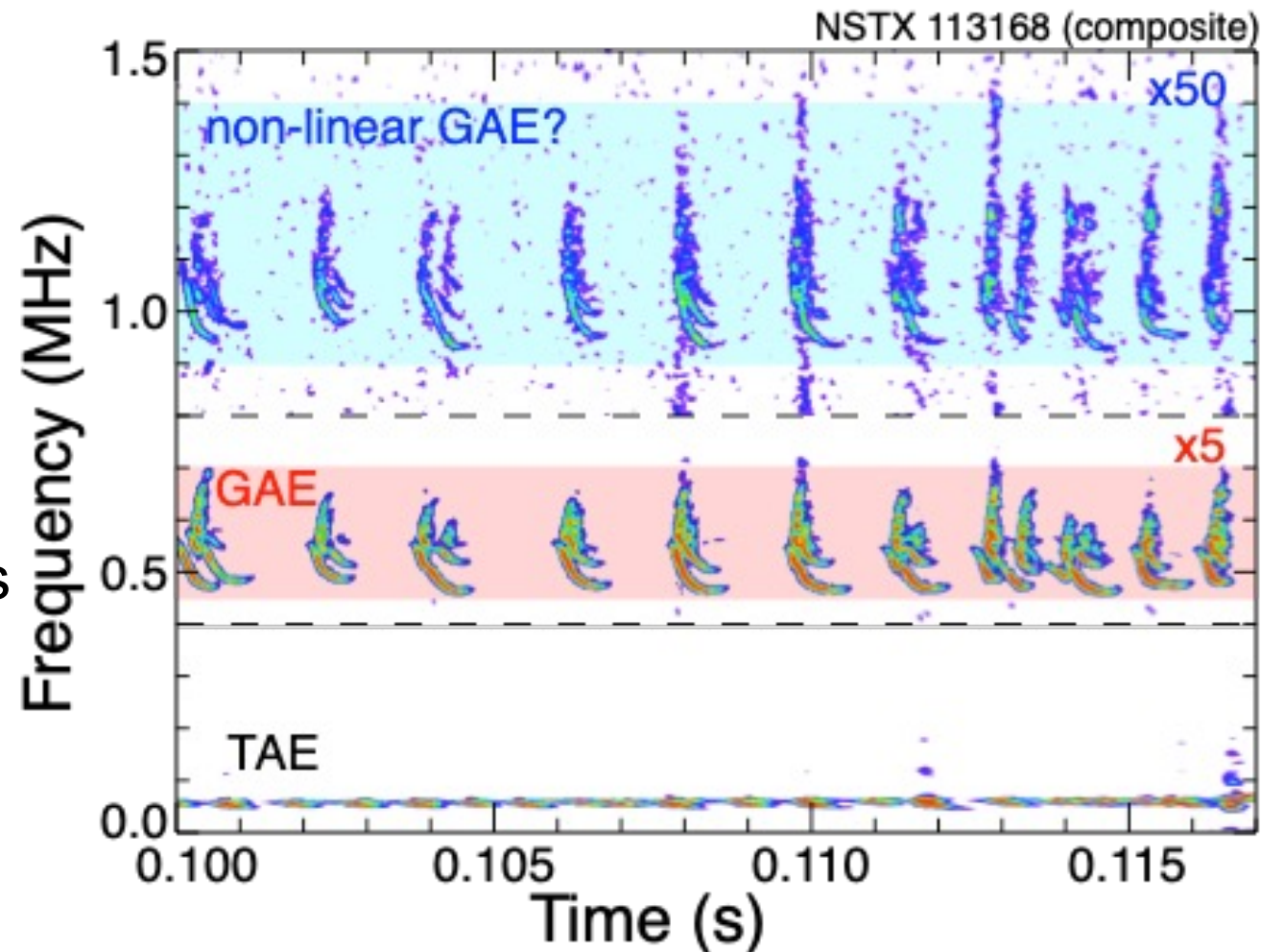
E. D. Fredrickson

EP Meeting  
PPPL, Princeton, New Jersey  
Sept. 14, 2022



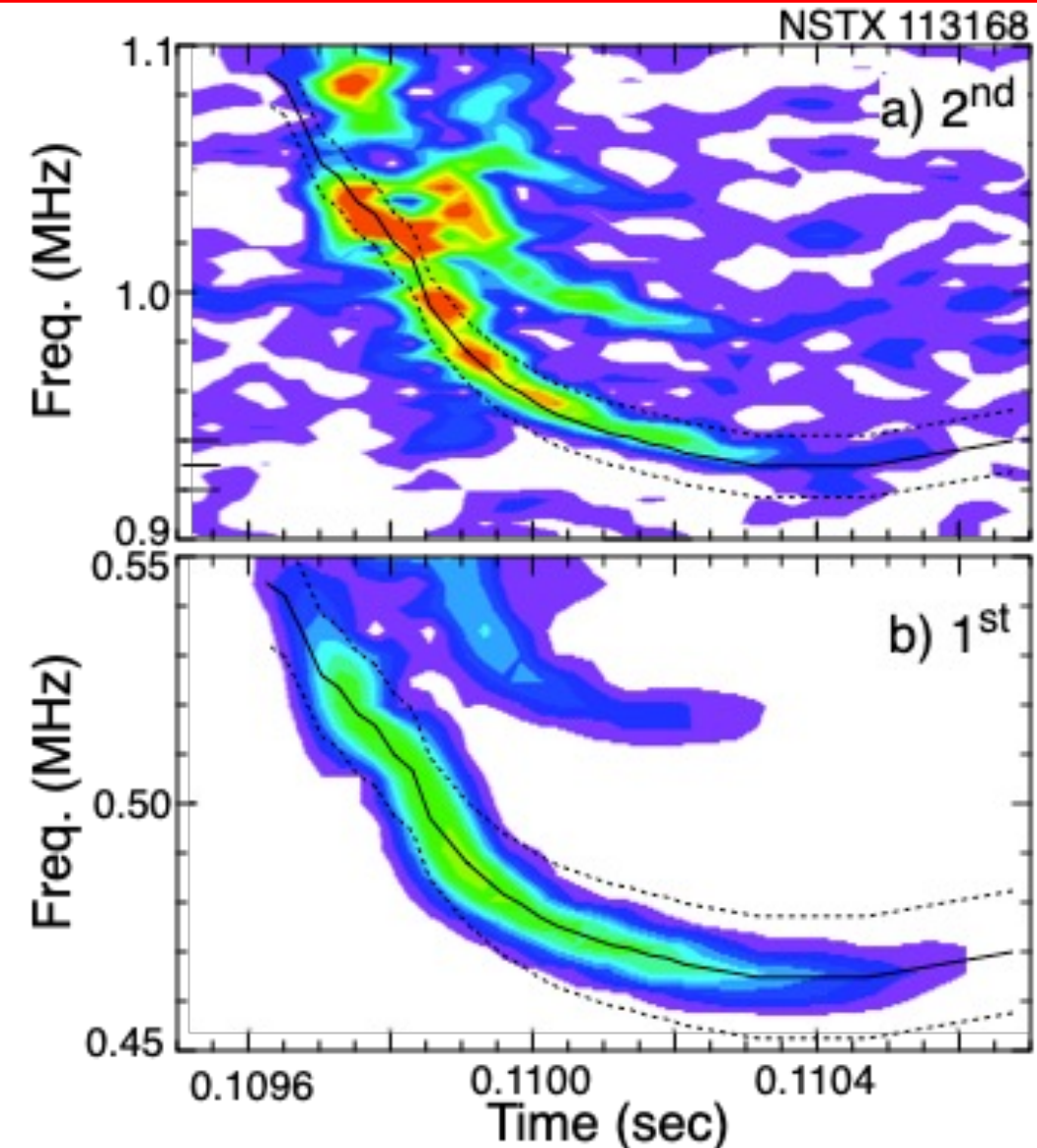
# Modes at the second harmonic of GAE are common in NSTX

- In this example, both 1<sup>st</sup> and 2<sup>nd</sup> harmonic GAE bursts show chirping consistent with formation of holes and clumps in the fast ion distribution
- 2<sup>nd</sup> harmonic GAE could be from:
  - non-linear terms in GAE dispersion equations => 3-wave coupling
  - independent weakly damped modes triggered by displaced fast ions
  - non-linearly driven GAE, off their natural frequency?
- Most observations support wave-physics non-linearities.
- Further, the data suggests in some cases that weakly damped modes were transiently excited.



# We begin with the GAE burst at 0.11s from previous slide, and its 2<sup>nd</sup> harmonic components

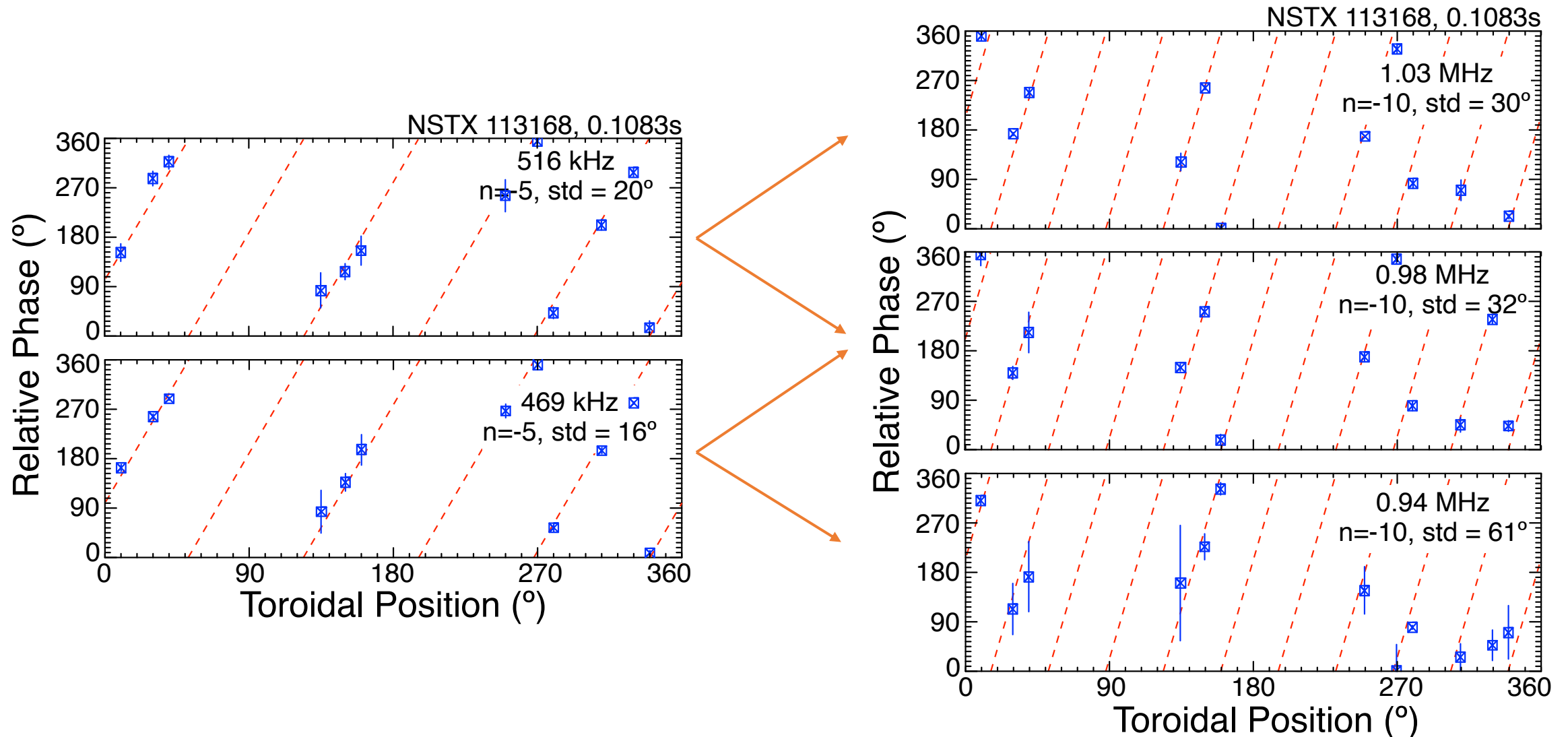
- A non-linear 2<sup>nd</sup> harmonic perturbations' amplitude should scale quadratically with the fundamental amplitude.
  - The frequencies should be the sum of frequencies of the fundamental mode
  - the mode number should be the sum of the fundamental mode numbers
- The amplitude scaling is found by tracking the mode frequency and calculating the RMS amplitude vs. time
  - solid line tracks mode center frequency.
  - short dashed lines indicate frequency range for RMS calculation.
- We compare the phase between the non-linear perturbation and 2<sup>nd</sup> harmonic mode.





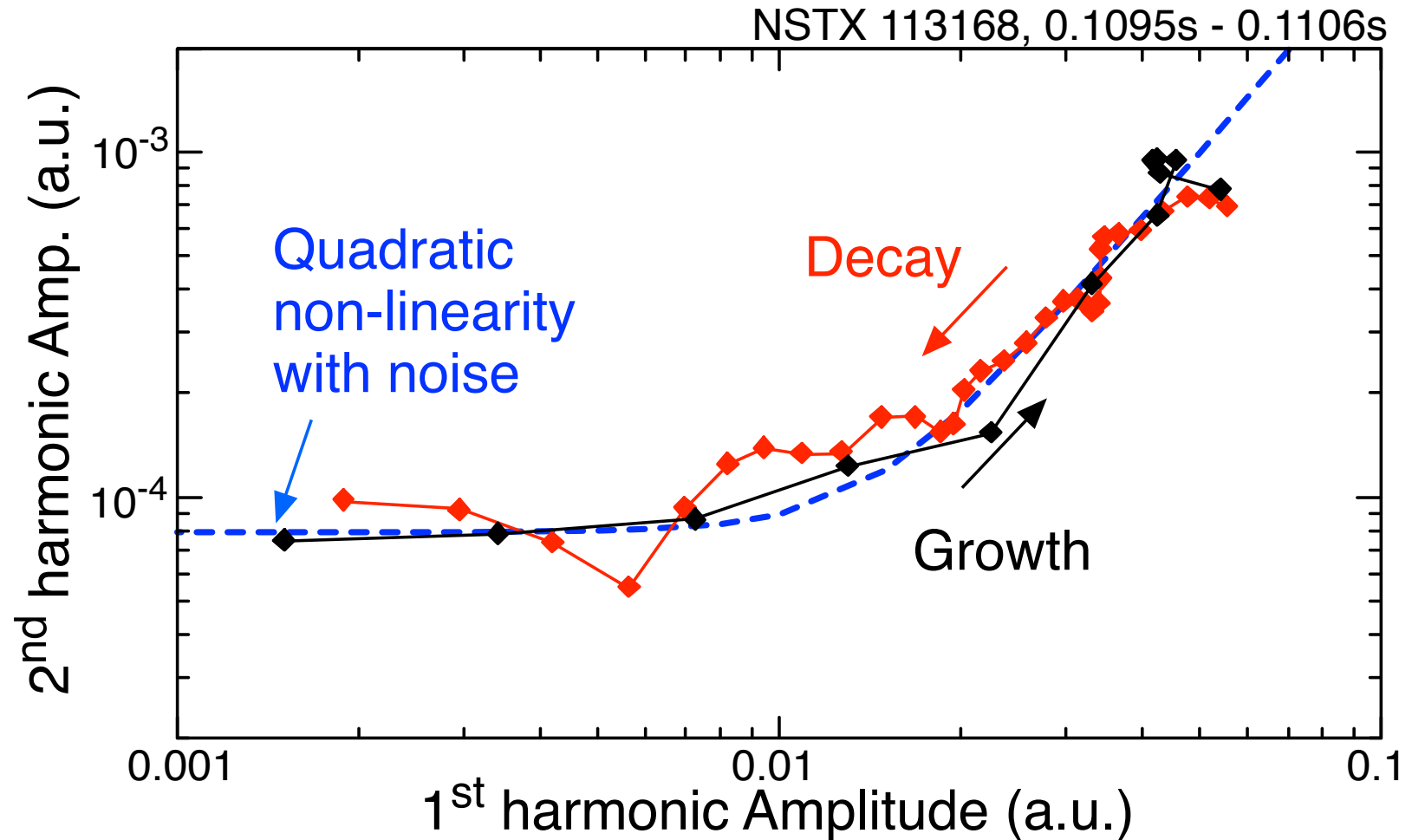
# In this example the 2<sup>nd</sup> harmonic frequencies and mode numbers are consistent with non-linear 3-wave coupling

- Fundamental modes were  $n = -5$  and 2<sup>nd</sup> harmonic modes were  $n = -10$ .



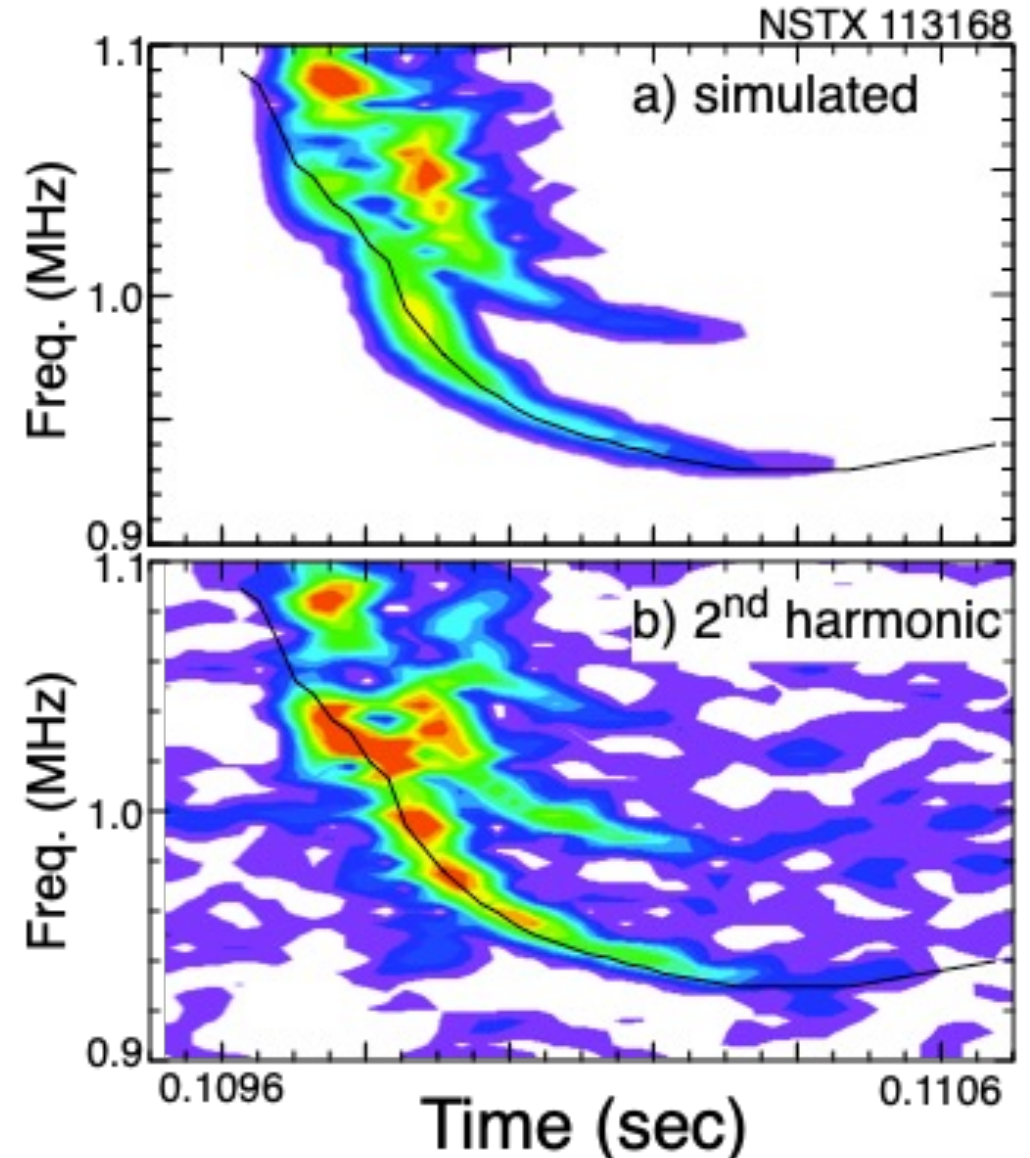
# 2<sup>nd</sup> Harmonic Mode amplitude tracks fundamental

- The time-dependent RMS mode amplitudes are calculated for a fundamental and 2<sup>nd</sup> harmonic GAE mode.
- The second harmonic amplitude is plotted vs. the first harmonic amplitude on a log-log scale.
- The weak 2<sup>nd</sup> harmonic mode has low signal-to-noise at the beginning and end of the pulse,
  - The blue dashed curve shows the expected trajectory of the 2<sup>nd</sup> harmonic amplitude including noise added in quadrature.



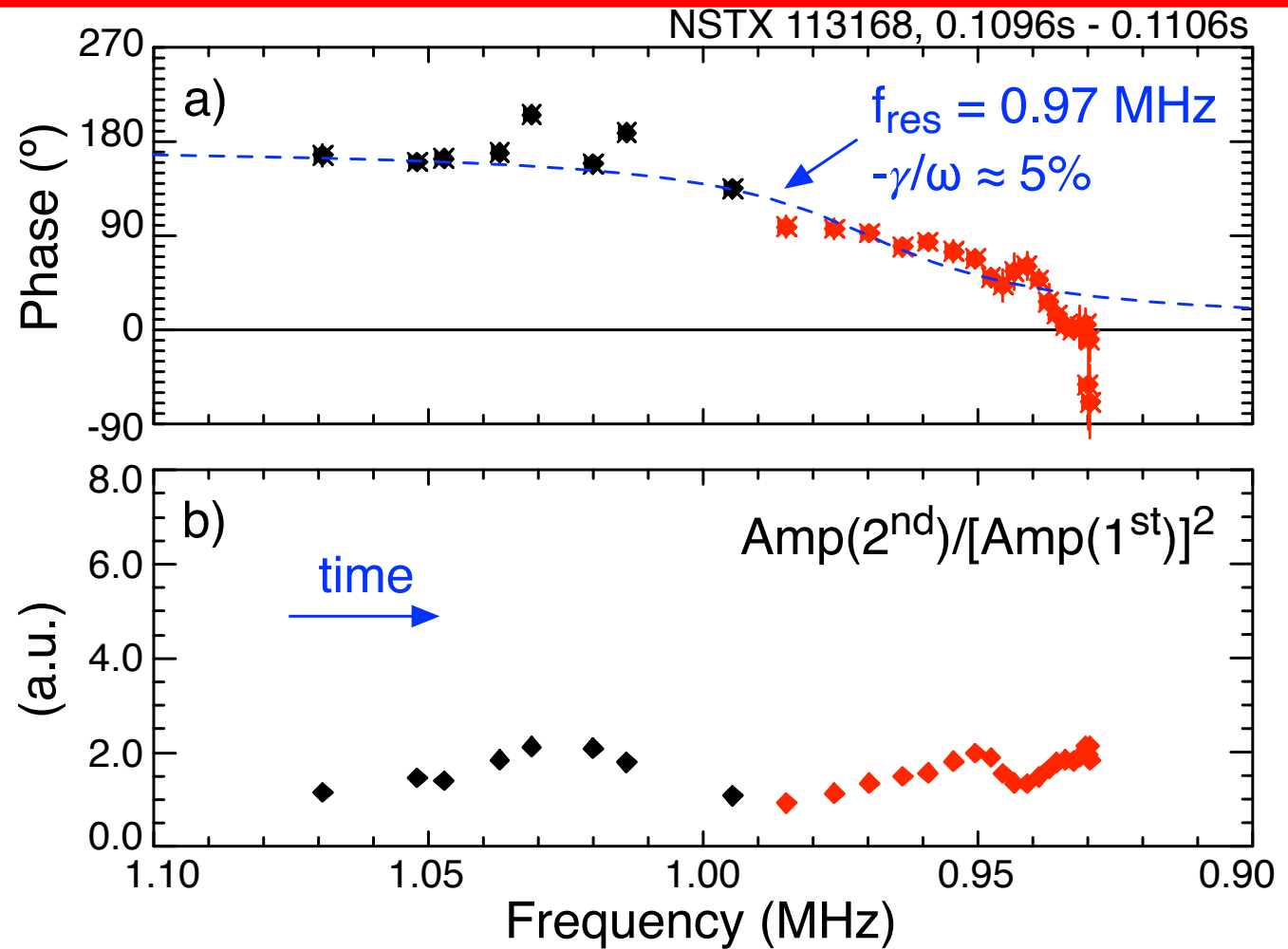
# We simulate the 2<sup>nd</sup> harmonic perturbations using the fundamental modes

- The raw data is filtered to preserve only the fundamental mode.
- That is squared, to simulate a quadratic, non-linear perturbation at the 2<sup>nd</sup> harmonic.
- The spectrograms of the simulated 2<sup>nd</sup> harmonic perturbation and the measured 2<sup>nd</sup> harmonic fluctuations are compared at right.
- There is a strong qualitative agreement between the two spectrograms,
  - There are some subtle differences in the relative mode amplitudes and in the amplitude evolutions,
  - although noise and potential differences in radial mode structure could account for some of this.



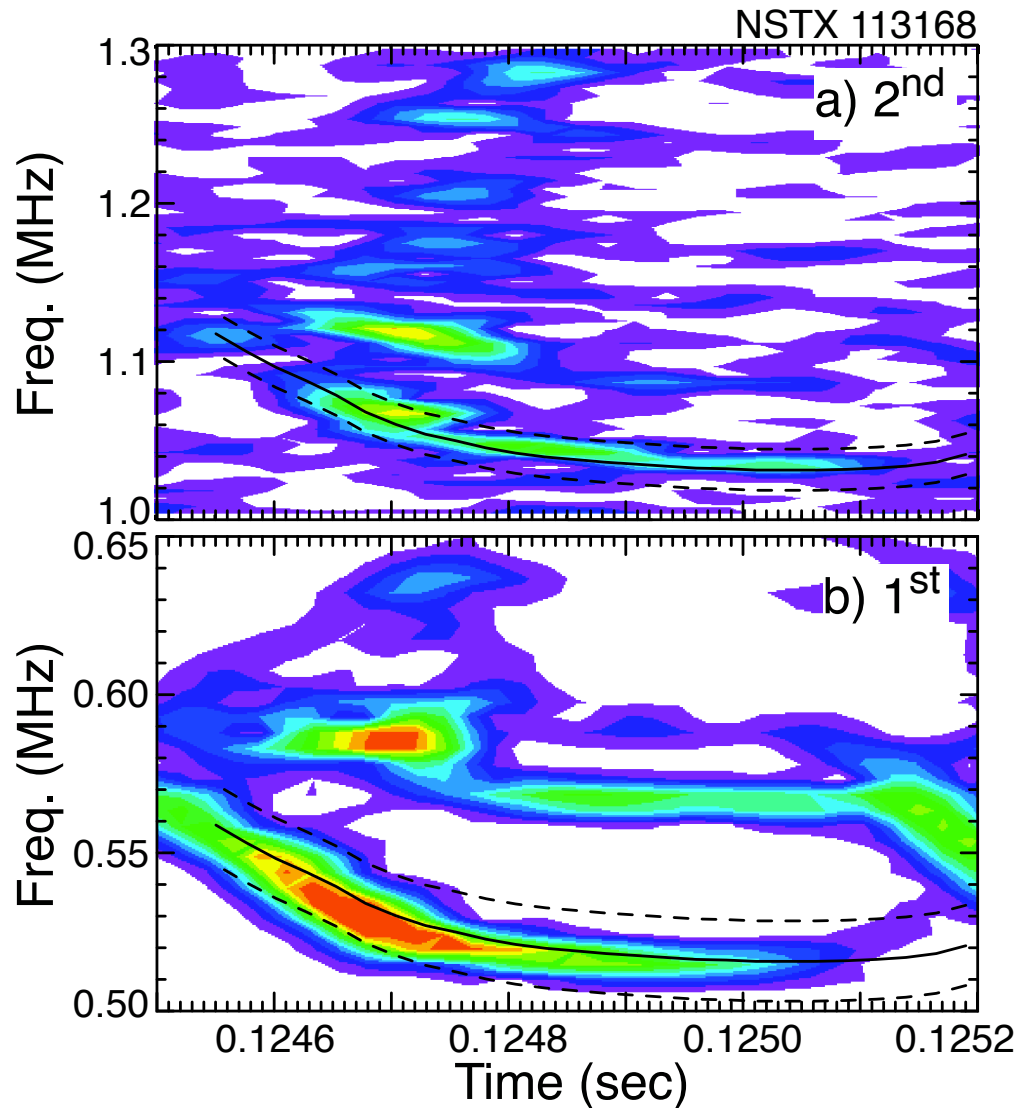
# The relative phase can be calculated between the simulated and measured 2<sup>nd</sup> harmonic modes

- The relative phase for the mode indicated by the black curve is shown at right.
  - The dashed blue line shows the (adiabatic) phase vs. frequency for a driven oscillator with damping.
- The 2<sup>nd</sup> harmonic amplitude is normalized to the square of the fundamental and shown in b),
  - The nearly constant ratio is consistent with a weak effect of the plasma on the 2<sup>nd</sup> harmonic perturbations.

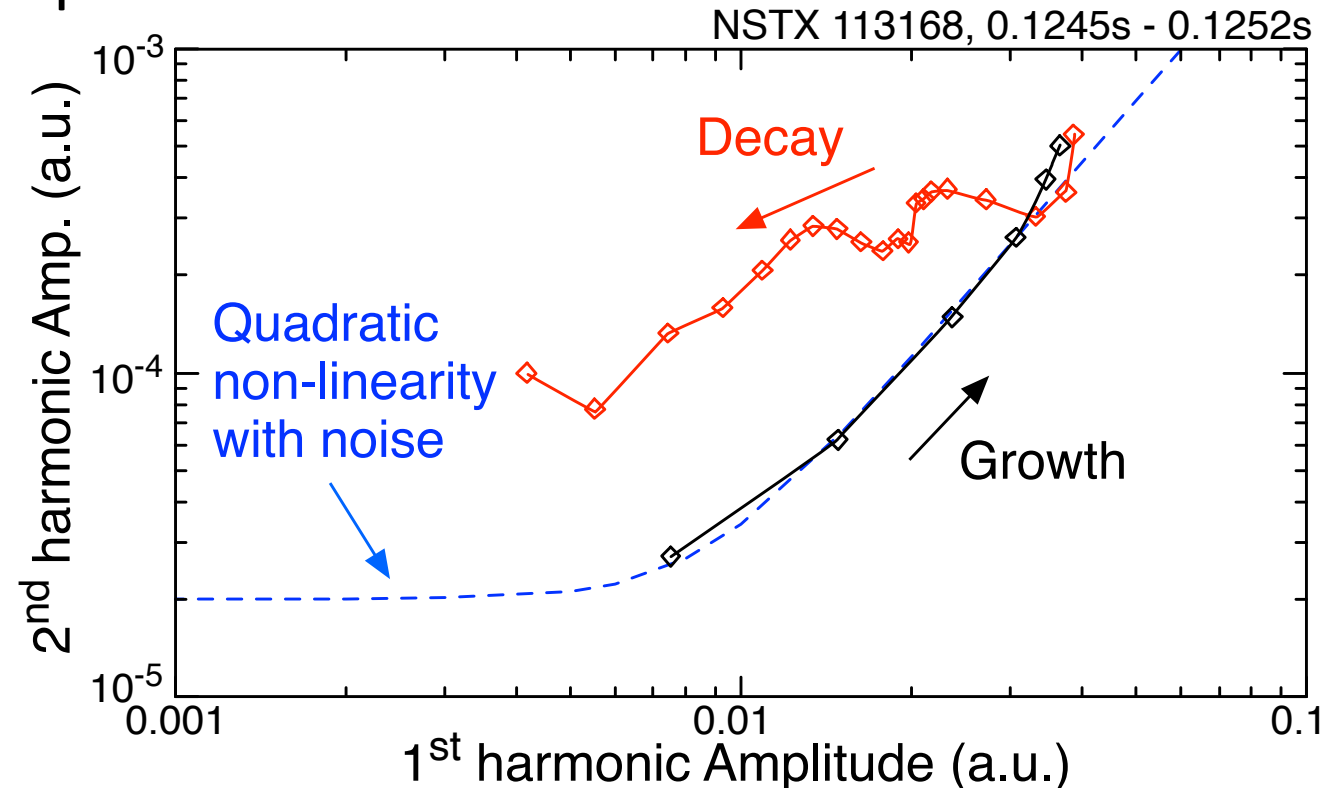


- Expected phase shift is  $\leq 180^\circ$  for drive frequency greater than resonant frequency,  $90^\circ$  at resonance and  $\approx 0^\circ$  for drive frequency less than resonant frequency.

# Other cases provide more evidence of the independence of the 2<sup>nd</sup> harmonic GAE

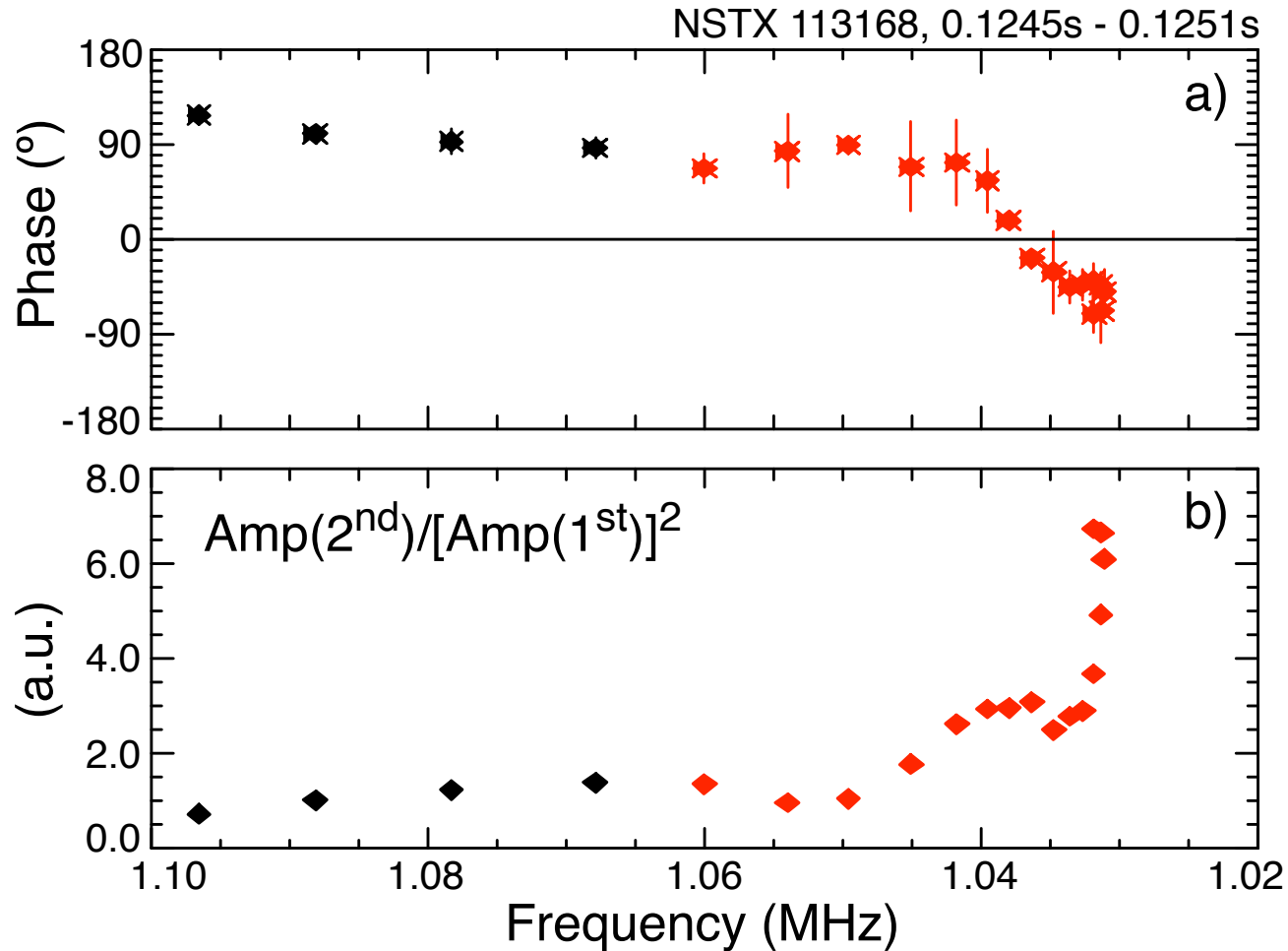


- Mode decay shows hysteresis, suggesting this is weak coupling to independent modes?
- This example is taken from same shot, but about 15 ms later – nearly identical plasma parameters.





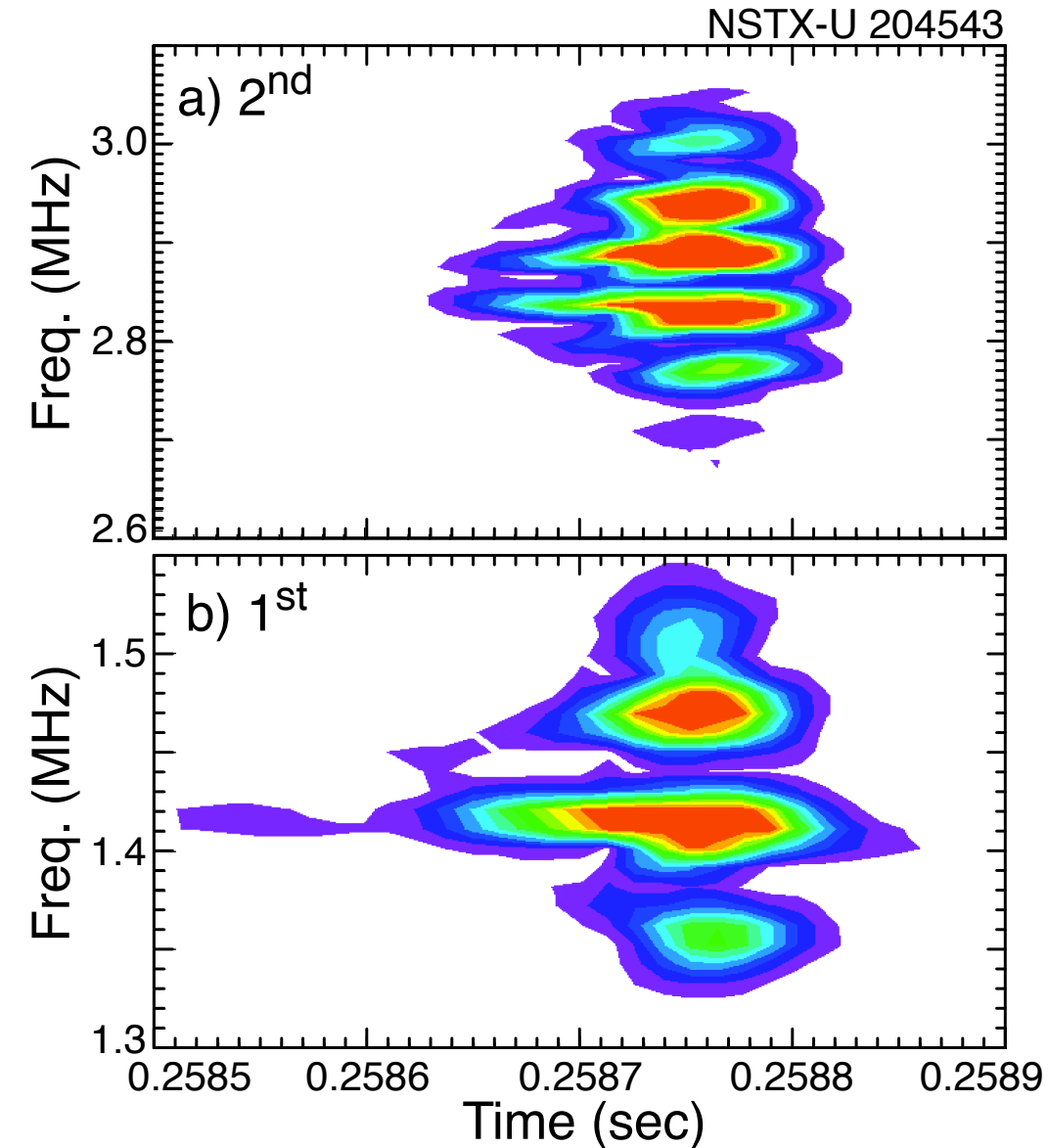
# The relative phase differs in this case



- The relative phase shift suggests drive frequency is near resonance frequency?
  - While frequency chirps down 60 kHz?
- Could damping be much higher in this shot, broadening resonance?

- Expected phase shift is  $\leq 180^\circ$  for drive frequency greater than resonant frequency,  $90^\circ$  at resonance and  $\approx 0^\circ$  for drive frequency less than resonant frequency.

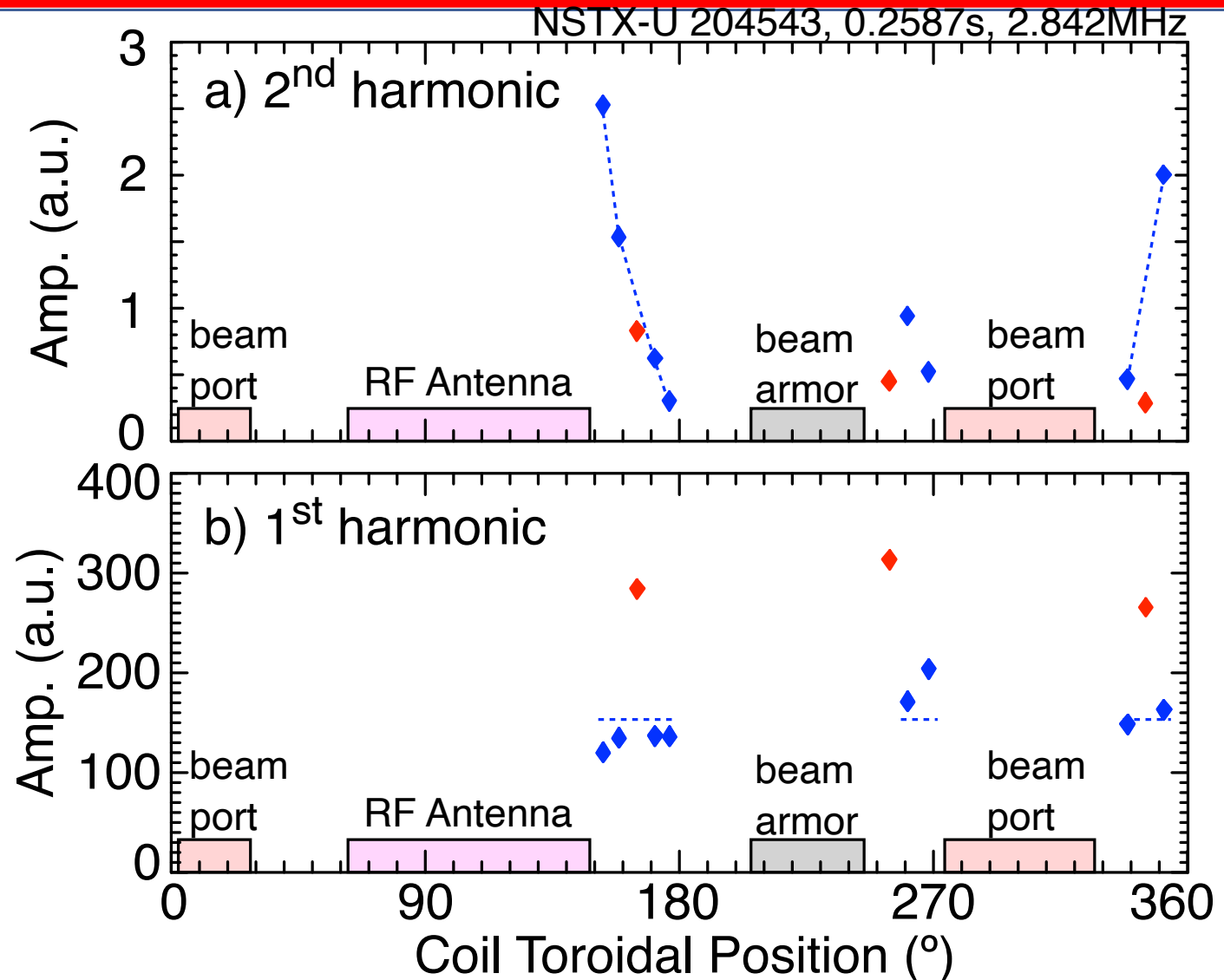
# GAE bursts in higher field plasmas are shorter, less likely to chirp



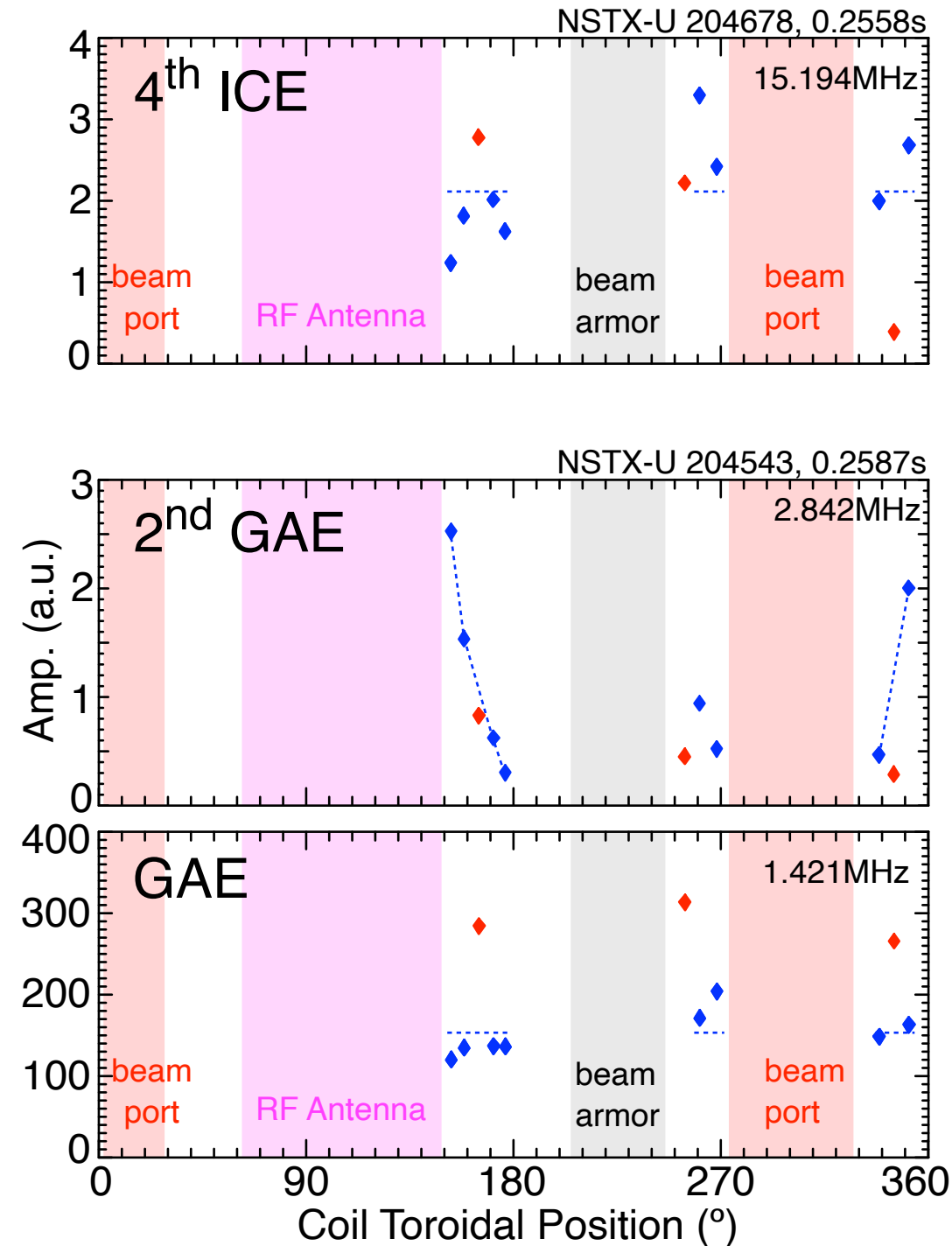
- A typical GAE burst in a 5.9 kG plasma
  - (previous example was 2.7kG).
- Fundamental mode numbers are  $n = 9, 10, 11$ .
- 2<sup>nd</sup> harmonics should range from  $n = 18$  to 22.
- There might be some small frequency chirping, but by any measure it's much less than in low field GAE bursts.

# Large toroidal variation in 2<sup>nd</sup> harmonic amplitude

- Data suggests 2<sup>nd</sup> harmonic mode is localized in front of RF antenna.
- Could the strong localization be correlated with the shorter wavelengths at higher field?
- Not much chance of adding sensors there.
- Plans are to add sensors in 30° to 60° span.



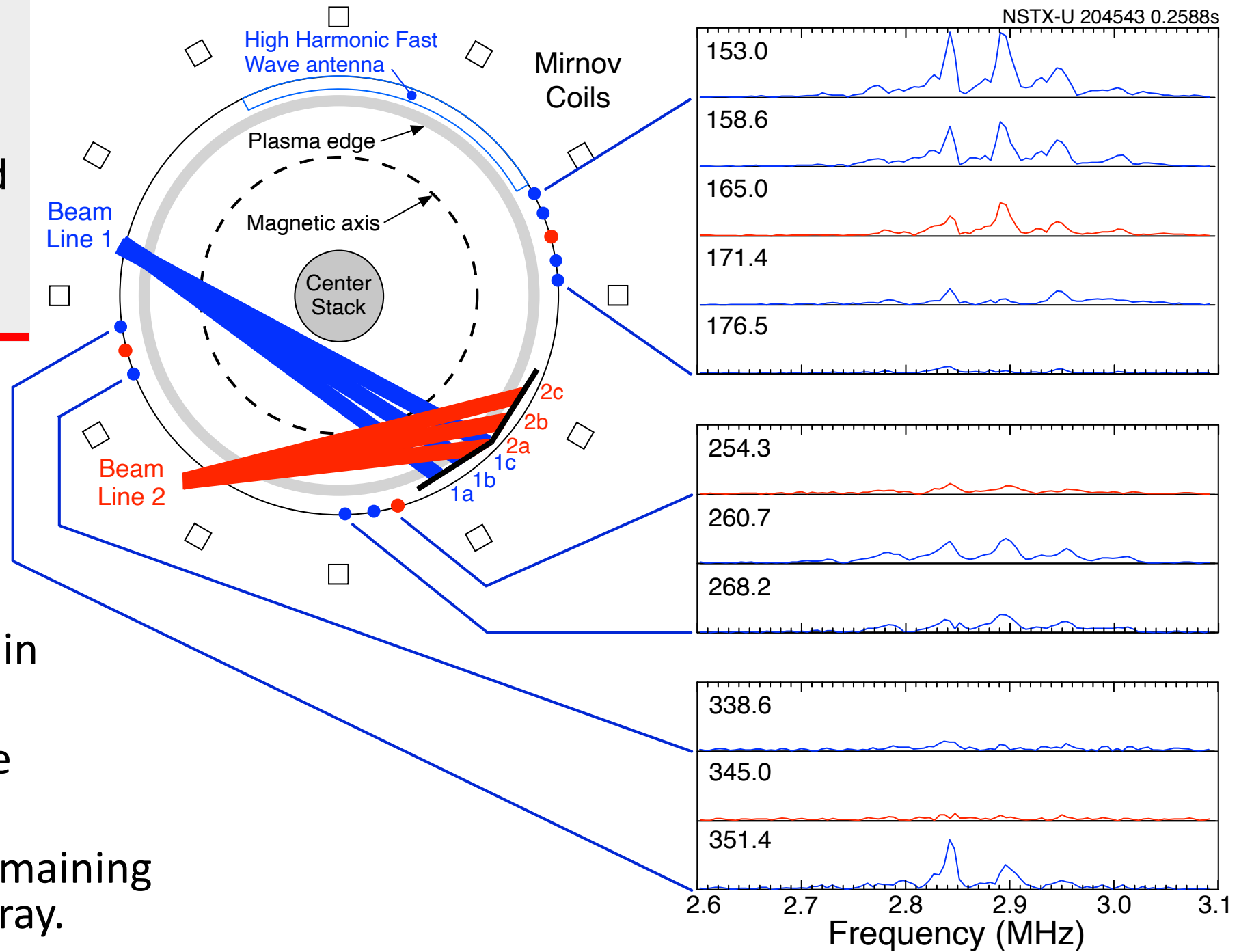
# Not a bandwidth issue for magnetic sensors



- Not much besides ICE inhabits the frequency range above 4 MHz in NSTX-U.
- The 4<sup>th</sup> harmonic ICE at 15 MHz is roughly toroidally symmetric,
  - it was also possible to measure the mode number.
- Generally, other ICE harmonics are also toroidally symmetric.
- ICE is also long wavelength with  $1 \leq |n| \leq 4$



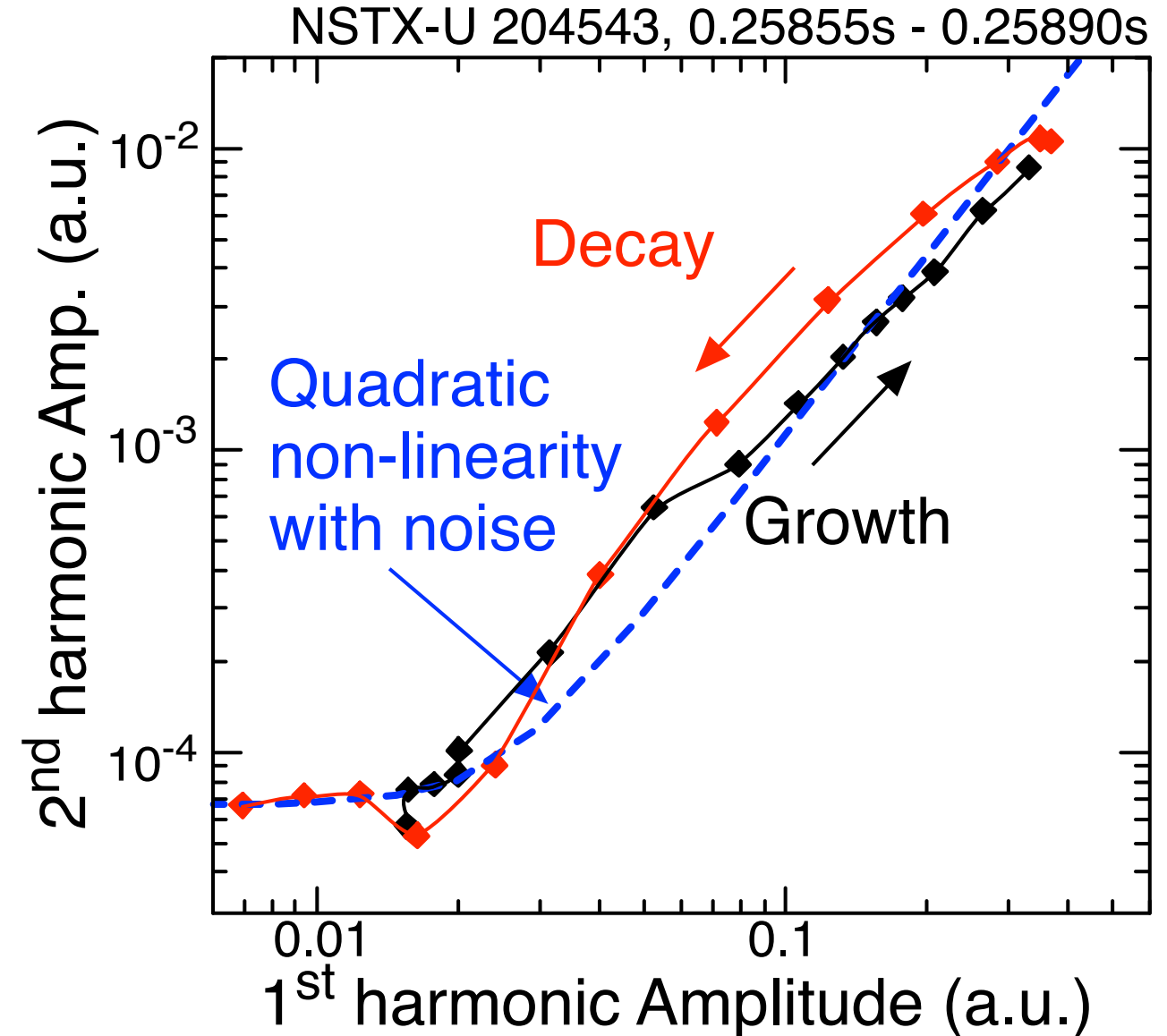
# Spectra show toroidal Amplitude of 2<sup>nd</sup> harmonic GAE



- Good signal-to-noise in spectra.
- All modes show some localization.
- Only limited space remaining to expand toroidal array.

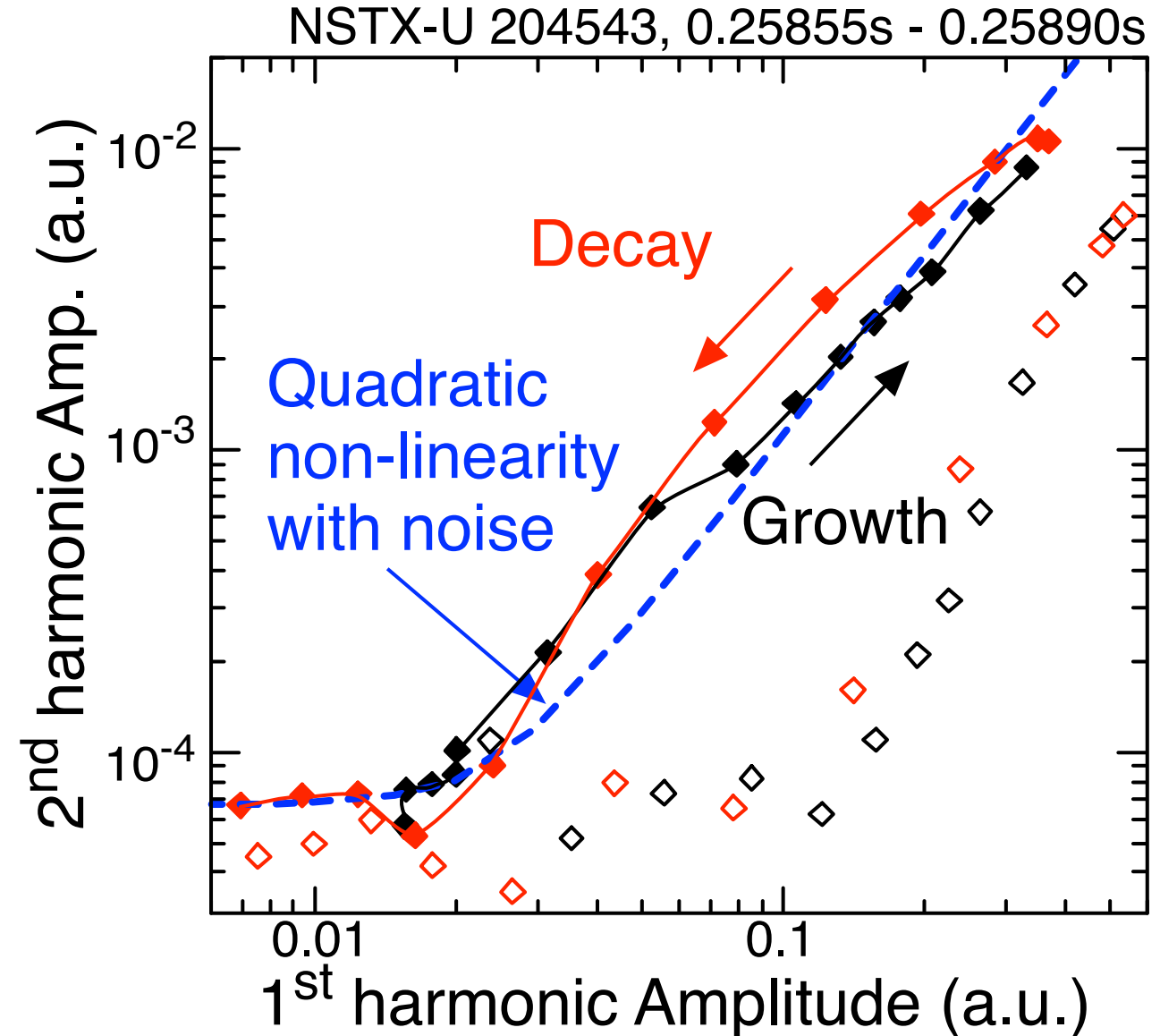
# Mode amplitudes approximately consistent with quadratic non-linearity

- Roughly quadratic ratio over two orders of magnitude
  - relatively lower noise in this frequency range.
- Scaling is same in strong and weak areas.



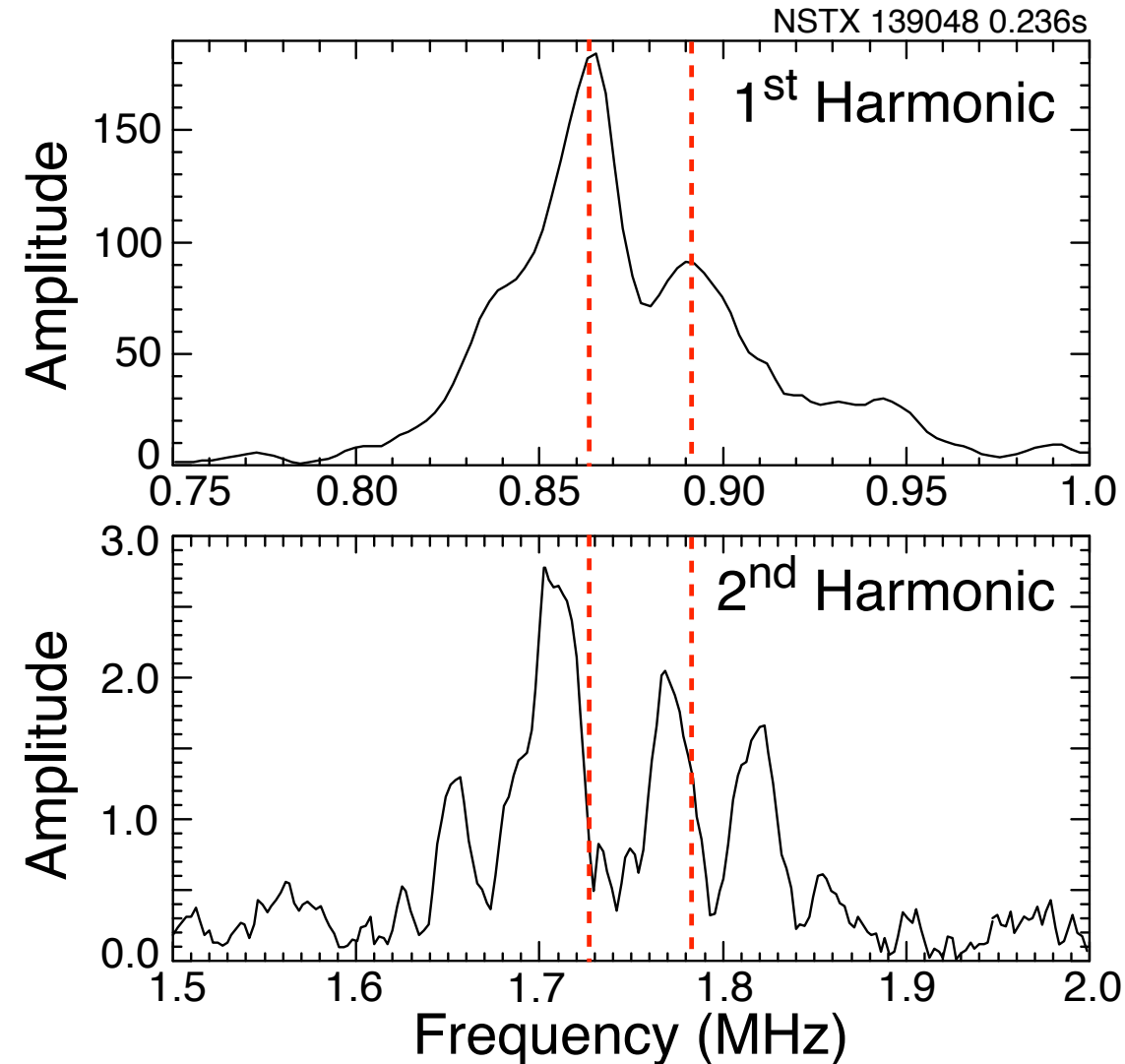
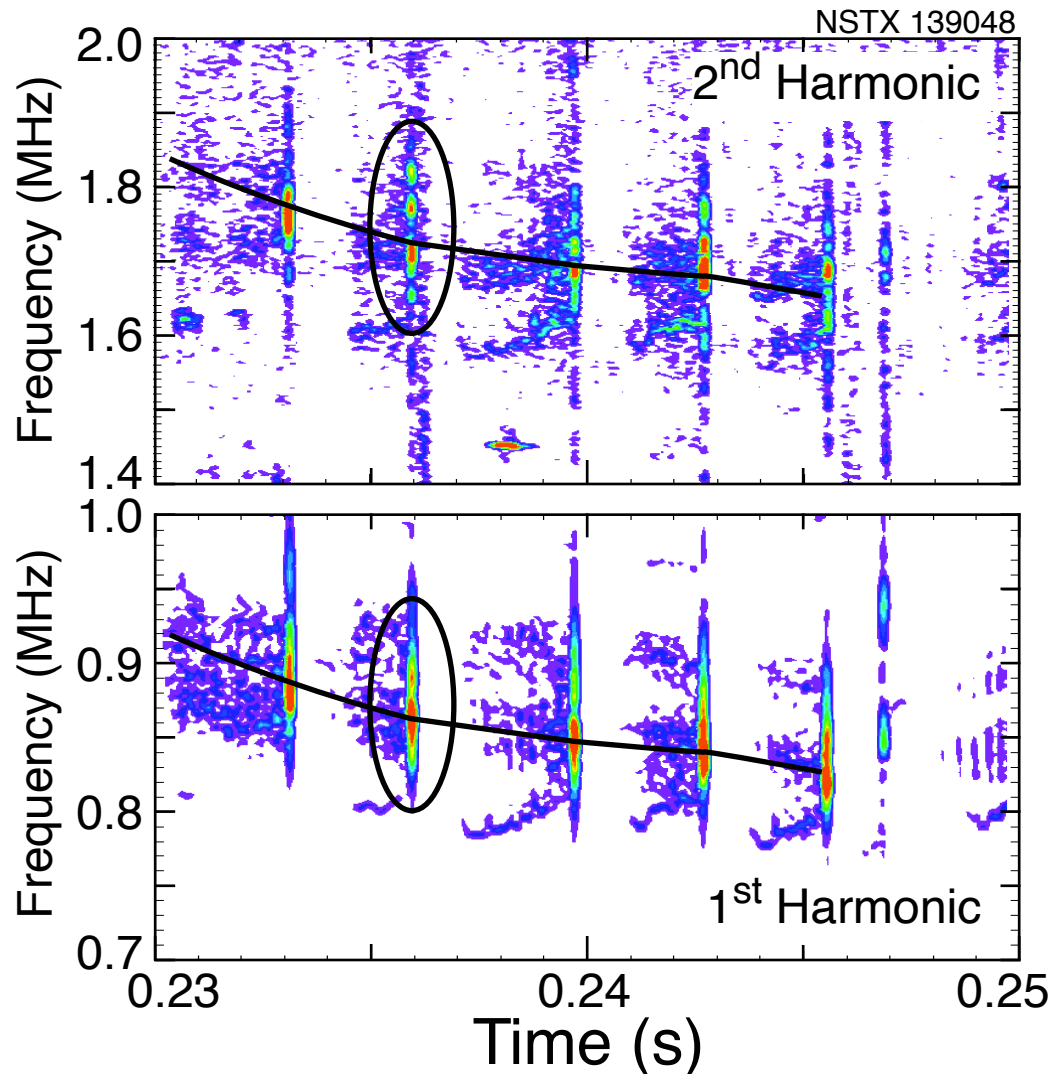
# Mode amplitudes approximately consistent with quadratic non-linearity

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- Scaling is same in strong and weak areas.



# Frequencies don't always satisfy 3-wave coupling rule

- However, noise from switching Power Amplifier precludes mode analysis of 2<sup>nd</sup> harmonic.
- This example also doesn't show frequency chirping.





# The 2<sup>nd</sup> harmonic GAE appear to be partially independent

- Amplitude evolution of 1<sup>st</sup> and 2<sup>nd</sup> GAE during burst appears to be de-coupled;
  - small caveat is that amplitude measurement is at plasma edge.
  - Possibly could be explained frequency-dependent 2<sup>nd</sup> harmonic damping term?
- Toroidally localized 2<sup>nd</sup> harmonic GAE inconsistent with simple non-linear response;
  - toroidal localization of short wavelength Alfvénic modes is something new.
  - Could have important ramifications for the modeling of TAE induced transport in ITER?
- These results suggest that the fundamental GAE are acting like an antenna where the non-linearity creates perturbations at twice the frequency and half the wavelength
  - not as much control over frequency as with an antenna, but wavelength is fixed vs. not well defined,
  - drive is weak, but has the advantage of being where the mode is, the antenna has to deal with fast fall-off,
  - because the excitation isn't from the edge, backing out the antenna drive from the measurement is easier,
  - Similar evidence of non-linearity has previously been seen with TAE.

# Future work:

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- Work out non-linear terms in the GAE dispersion relation derived from Brazinski 2-fluid;
  - is relative amplitude of harmonics reasonable?
  - can anything be understood from scaling with plasma parameters?
- Would hole-clump model of chirping predict non-linear terms as a result of driving GAE off their natural resonance?
  - Fundamentally, waves transfer energy between kinetic (motion of plasma tied to field lines) and potential (energy stored in perturbed magnetic field geometry).
  - If waves driven off natural frequency, energy must be stored in fast-ion distribution to compensate for lower kinetic energy from slower motion at lower frequencies. This would introduce perturbations at twice the mode frequency.
- Larger collection of data needs to be analyzed.
  - How common are disparate frequencies? Disparate mode numbers?
  - What about hole-clump chirps at higher field? Where are they?
  - “Good” Mirnov data only available from 2016 campaign (SPA noise), which was all done at  $\approx 6\text{kG}$ .